Kuwait University	Math. 101	January 6th, 1993
Mathematics Dept.	Final Examination	Duration: 2 hours

Answer the following questions. CALCULATORS ARE NOT ALLOWED.
DO NOT INCLUDE MORE THAN ONE SOLUTION FOR EACH QUESTION.

1. (a) Find the equation of the circle whose radius is 4, tangent to the x-axis and above it and has its center on the line $x - 2y - 2 = 0$.

   (b) Find the point(s) on the graph of $y^2 + x = 0$ that is closest to the point $(-3, 0)$.

(6 points)

2. Evaluate the following limits, if they exist,

   (i) $\lim_{x \to 1} \dfrac{\sqrt[3]{x} - 1}{x - 1}$ (ii) $\lim_{x \to -\infty} \dfrac{\sqrt[3]{8x^3 + 1}}{|x|}$

(6 points)

3. Evaluate the following integrals

   (a) $\int (5 + \cos x)^3 \sin x \, dx$ (b) $\int_0^2 |x^3 - x^2| \, dx$

(6 points)

4. (a) Use differentials to find an approximate value of $\sqrt{(3.02)^3 - 2}$

   (b) Show that $\tan x > x$, for $0 < x < \frac{\pi}{2}$

(6 points)

5. (a) Let y be a function of x implicitly defined by $3x^2 - xy^2 + 2y = 12$. Find the equation of the normal line to the graph of y at the point $(2, 0)$.

   (b) Find the constants A and B so that

$$f(x) = \begin{cases} 2\sin x + \cos x & \text{, if } x < 0 \\ Ax + B & \text{, if } x \geq 0 \end{cases}$$

   is continuous and differentiable at $x = 0$.

(8 points)

6. (a) Find a continuous function f and a real number a such that

$$\int_a^x 2f(t)dt = -1 + 2\sin x, \qquad \text{for } -\infty < x < +\infty.$$

   (b) Find the average value of the function $f(x) = 1 + \sqrt{4 - x^2}$, on the interval $[-2, 2]$.

(8 points)

7. Let $f(x) = \dfrac{3x^2 - 4x - 4}{x^2}$

   (a) State the domain of f.
   (b) Find the x and y intercepts, if any.
   (c) Find the vertical and horizontal asymptotes of f , if any.
   (d) Find the intervals on which f is increasing and the intervals on which f is decreasing.
   (e) Find the local extrema of f, if any.
   (f) Determine where the graph of f is concave upward and where it is concave downward.
   (g) Find the inflection point(s) of f , if any.
   (h) Sketch the graph of f using the information in (a)–(g).

(10 points)